

*An analytical approach to activating  
demand elasticity  
with a demand response mechanism*

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# 1. Introduction

- Smart grids technologies will deeply modify distribution and final consumers' environment.
- Consumers could now receive several signals (informations, prices) to adapt their behaviours.
- This is a new step in electricity markets as demand is often seen as inelastic.
- However, consumers' fears of greater electricity bills increases with the use of Demand Response (DR) tools (Herter, 2007; Park et al., 2014)
  - => A DR design “that benefits a few at the expense of many has little chance of customer acceptance and regulatory approval” (Herter, 2007).

# Some examples of signals and load reductions

- Indirect feedback (education, information campaigns)
  - => Few impacts on consumptions (0 to 7% of load reduction).
- Direct feedback (In Home Display, monitoring datas from smart meters)
  - => More impacts (2 to 15% of load reduction).
- Dynamic pricing (with or without Direct Load Control)
  - => The greatest impact on load reduction (5 to 50% of load reduction for some periods).

## 2. Literature

- Lijensen (2007) => Consumers of electricity are captives in the short run.
- Haney and al. (2009); Faruqui and Sergici (2010) => Elasticity of demand could appear with SG technologies and DR programs.
- Herter (2007) => Consumers could be worse off with DR mechanism (dynamic pricing Critical Peak Pricing).
- Léautier (2014) => Marginal value of Real Time Price decreases with the number of consumers at RTP.

# 3. Motivations

## Objectives

1. To study the impact of demand response programs in interconnected markets on consumers' surplus and welfare.

## Approach

We use a deterministic optimization model with supply functions and market constraints (Vespucci et al., 2013).

## Main results

1. There is an optimal level for the price signal at which DR increases social welfare or consumers' surplus.
2. This level is negatively correlated to the degree of competitiveness of the generating technologies and the size of the market.
3. Unserved energy would limit DR and this constraint is greater if energy trade between countries is limited.

## 4. The Model : assumptions (1)

- Two interconnected markets ( $\{n \neq m\} = \{1, 2\}$ ) with an interconnection capacity  $Cap_{n,m}$  at price  $P_{n,m}$ .
- $k$  technologies in each country,  $t = \{1, \dots, k\}$ .
- Each technology has a variable cost function :

$$CV_{t,n}(x_{t,n}) = a_{t,n} \cdot x_{t,n} + 1/2 \cdot b_{t,n} \cdot x_{t,n}^2$$

- For simplicity's sake we shall rank the technologies from the least to the most expensive

$$a_{t,n} = \alpha^{t-1} \cdot a \text{ and } b_{t,n} = \alpha^{t-1} \cdot b$$

With  $a$  and  $b$  the cost parameters of the least expensive technology and  $\alpha > 1$ .

## The Model : assumptions (2)

- A system operator builds an aggregate supply function with all suppliers' bids :

$$S_{\downarrow n}(X_{\downarrow n}) = a_{\downarrow n} + b_{\downarrow n} X_{\downarrow n}$$

$$\text{With } X_{\downarrow n} = \sum_{t=1}^T \sum_{k \in K} (x_{\downarrow t,n} + x_{\downarrow t,n,m}).$$

- As in others papers (Woo, 1990; Stoft, 2002; De Jonghe et al., 2011), we assume perfect competition between suppliers (i.e bids are made at marginal cost).

## The Model : assumptions (3)

- Two scenarios of demand with and without DR technologies :

- With no DR technologies, an inelastic demand function  $\bar{D} \downarrow n$  with a final consumption  $Q \downarrow n = \bar{D} \downarrow n$  .
- With DR technologies, demand becomes elastic

$$D \downarrow n \uparrow^{-1} (X \downarrow n) = c \downarrow n - d \downarrow n \cdot X \downarrow n$$

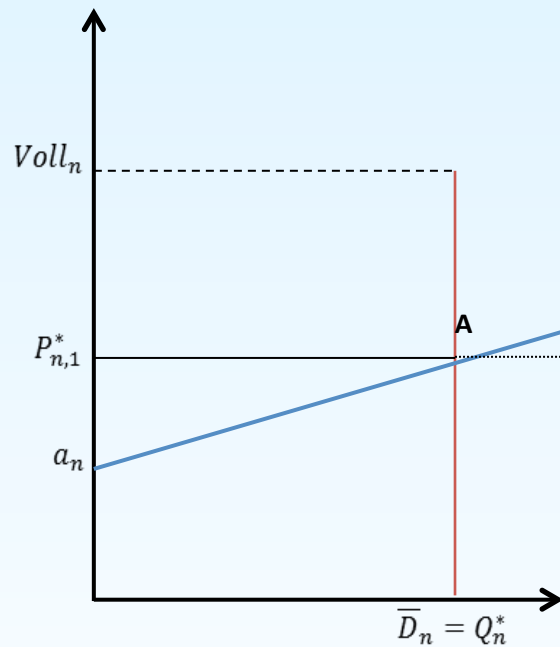
With a final consumption  $Q \downarrow n = D \downarrow n \uparrow^{-1} (S \downarrow n (X \downarrow n))$ .

- DR technologies are costly thus it induces an increase in operational costs of  $1/2 \gamma \downarrow n \cdot x \downarrow t, n \uparrow^2$  where  $\gamma \downarrow n$  can be interpreted as a DR intensity parameter.

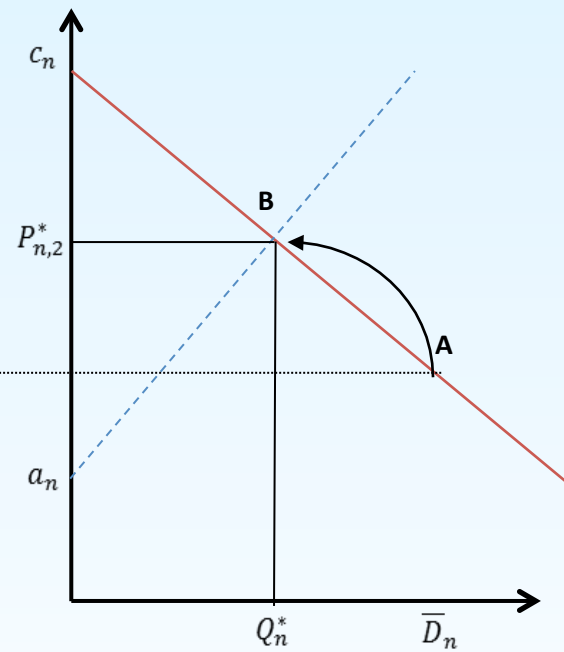


# The Model : two scenarios of demand

Market equilibrium in a given country without Demand Response



Market equilibrium in a given country with Demand Response



## Baseline case : Welfare analysis with no DR (1)

- System operator maximises the welfare to compute market equilibrium :  $Q \downarrow n = \sum D \downarrow n$

$$\text{Max} \int_0^T (V \downarrow n - S \downarrow n (X \downarrow n)) dX \downarrow n$$

$$\text{Subject to : } \begin{cases} x \downarrow t, n + x \downarrow t, n, m \leq K \downarrow t, n & (r \downarrow t, n) \\ \sum t \uparrow x \downarrow t, n, m \leq \text{Cap} \downarrow n, m & (P \downarrow n, m) \\ \sum t \uparrow x \downarrow t, n + \sum t \uparrow x \downarrow t, m, n = Q \downarrow n & (f) \end{cases}$$

- The Nash equilibrium minimizes the variable costs of generation in each country.

## Baseline case : Welfare analysis with no DR (2)

- Efficient technology  $t$  would offer all its installed capacity :

$$x_{t,n}^* = K_{t,n} / 2 + Q_{n,m} / 2 \cdot \omega_{t,n} + \sum_{n,m} P_{n,m} \cdot \sum_{t,m} 1 / \alpha_{t,m} / b \cdot \omega_{t,n}$$

$$x_{t,n,m}^* = K_{t,n} / 2 - Q_{n,m} - Q_{n,m} / 2 \cdot \omega_{t,n} - \sum_{n,m} P_{n,m} \cdot \sum_{t,m} 1 / \alpha_{t,m} / b \cdot \omega_{t,n}$$

- Inefficient technology  $\bar{t}$  would serve the residual demand:

$$x_{\bar{t},n}^* = Res_{n,m} - \phi_{\bar{t},n} / \omega_{\bar{t},n} + P_{n,m} / b \cdot \omega_{\bar{t},n} \cdot \sum_{t,m} 1 / \alpha_{t,m}$$

$$x_{\bar{t},n,m}^* = Res_{n,m} - \phi_{\bar{t},n} / \omega_{\bar{t},n} - P_{n,m} / b \cdot \omega_{\bar{t},n} \cdot \sum_{t,m} 1 / \alpha_{t,m}$$

## Baseline case : Some intuitive analysis

- If  $Q \downarrow n \geq Q \downarrow m$ , the production which meets local demand  $x \uparrow^* \downarrow t, n$  is far higher than the quantity exported  $x \uparrow^* \downarrow t, n, m$ . In this case, the opportunity cost of selling energy to the local market is higher than exporting.
- The greater the efficiency of technology  $t$ , i.e  $\omega \downarrow t$  decreases, the more it will be preferred for balancing the system of the country with higher energy consumption.
- To maximize social welfare, it would be beneficial for country  $m$  with inefficient technologies to rely more on imports even with substantial interconnection prices, leading to an increase

## 5. Demand response analysis (1)

- We assume a cost-efficient country scenario and a cost-inefficient country scenario.
- $SW_{\downarrow n,1} = \int_0^{\uparrow D_{\downarrow n}} (Voll_{\downarrow n} - S_{\downarrow n}(X_{\downarrow n})) dX_{\downarrow n}$  : social welfare at equilibrium before deploying the DR scheme.
- $SW_{\downarrow n,2} = \int_0^{\uparrow Q_{\downarrow n}^*} (D_{\downarrow n}^{-1}(X_{\downarrow n}) - S_{\downarrow n}(X_{\downarrow n})) dX_{\downarrow n}$  : social welfare at equilibrium after deploying the DR scheme.
- We define  $Q_{\downarrow n,min}$  as the minimum consumption above which social welfare does not decrease.
- Thus  $Q_{\downarrow n,min}$  is a threshold such as

## Demand response analysis (2)

- Solving  $\Delta \downarrow n \geq 0 \Leftrightarrow Q \downarrow n \uparrow^* \geq Q \downarrow n, \min = 2 \cdot SW \downarrow n, 1 / c \downarrow n - a \downarrow n$  ;
- As long as the equilibrium  $Q \downarrow n \uparrow^* \geq Q \downarrow n, \min$  , DR program improves the welfare.
- Supply and demand equilibrium leads to  $Q \downarrow n \uparrow^* = c \downarrow n - a \downarrow n / \gamma \downarrow n + b \downarrow n + d \downarrow n$  .
- Intuition : if  $\gamma \downarrow n$  is important, DR could be detrimental for welfare.
- Intensive DR should have a greater impact reducing both consumer surplus and the profits of firms.

# Efficient country and demand response (1)

- We discuss on the level of  $Q_{n,min}$  that is the minimum demand from consumers to keep gains in welfare.

- According to equilibria on supply markets, we could compute

$$\sum_{t \in T} K_{t,n} - \alpha \cdot Q_{n,m} + P_{n,m} / (\alpha - 1) \cdot b / 2 - \alpha = 2 \cdot SW_{n,1} / (c_{n,1} - a_{n,1})$$

- We use three parameters to analyse this relationship :
  - The market size ( $\sum_{t \in T} K_{t,n}$  and  $\bar{D}_{n,1}$ );
  - The level of cost efficiency of technologies ( $a_n$  and  $b_n$ );
  - The limitation on transmission capacity.

## Efficient country and demand response (2)

- The market size :
  - A large size increases  $Q \downarrow n, min$ .
  - Intensive DR is also constrained because of the impact on producers of the value of non-served energy.
  - Welfare losses could occur because of the cost of rationing consumers (and the voluntary increase in costs/prices by  $\gamma \downarrow n$ ) whereas generation is cheap.



## Efficient country and demand response (3)

- The efficiency of technologies :
  - If technologies are increasingly efficient, we observe an increase in  $Q_{\downarrow n, \min}$ .
  - Intensive DR would entail a higher opportunity cost which in turn would limit DR efficiency.
- The transmission capacity
  - A high  $P_{\downarrow n, m}$  increases  $Q_{\downarrow n, \min}$ .
  - Exports are costly for cheaper technologies thus the efficiency of the system decreases.

# Efficient country and demand response (4)

- Some conclusions :
  - DR could reduce the welfare for efficient technologies;
  - A high level of “inelastic” demand or of generation capacity increases the occurrence of welfare losses;
  - An aggressive DR scheme could only be deployed if  $b \downarrow n$  was high and  $\bar{D} \downarrow n$  low.

# Inefficient country and demand response

- Some conclusions using welfare analysis:
  - Costs parameters and the level of “inelastic” demand have the same effect on  $Q_{\downarrow n, min}$  as previously;
  - The intensity of DR is a decreasing function of efficient imports;
  - Transmission capacity price increases the intensity of DR;
  - An aggressive DR scheme could also be deployed if  $b_{\downarrow n}$  is high and  $K_{\downarrow t, m}$  is low.

# Demand response and consumers' surplus

- More aggressive DR could be adopted if :
  - Demand is high : consumers could easily reduce their demand;
  - Inefficient technologies : consumers' bills are high thus they accept deeper reductions in their consumption.
- Same impact of transmission capacity prices as in the welfare analysis.

## Demand response and elasticity

- A DR scheme is efficient if  $\varepsilon \downarrow n \geq 1 - c \downarrow n / d \downarrow n$  .  
 $Q \downarrow n, \min$  .
- The more efficient the system is, the more elastic consumer-demand must be, to sustain DR efficiency.
- Higher elasticity of demand is needed to compensate for the loss of producer surplus induced by the DR scheme in a welfare analysis.

# Conclusion

- Activating the elasticity of consumer-demand could benefit social welfare.
- In interconnected markets, trades, opportunity costs of energy, market size and efficiency of technologies are key parameters.
- Under consumers' surplus criteria, more aggressive DR could be adopted, the weight of producer welfare being removed.
- The impact on producers' surplus must be considered as a constraint on deploying demand response.
- Further improvements : strategic interaction between producers, consumers' behaviours (utility function, type and consumption profiles).

# Thank you for your attention

