

The Origins of the U-Shaped Average Cost Curve: Understanding the Complexities of the Modern Theory of the Firm

Jan Horst Keppler and Jérôme Lallement

Abstract

Presenting the history of the U-shaped average cost curve, a key element of microeconomic theory, this article fulfils a triple purpose. First, it establishes primacy of contribution and clarifies a common misunderstanding: Barone, Edgeworth and Sraffa share the development of a concept habitually attributed either to Marshall or Viner. Second, it shows that the complications surrounding the concept, its history and attribution are not coincidental. The equilibrium of a discretely-sized firm in a competitive industry is an ambivalent compromise between logical consistency built around a long-run notion of constant costs, on the one hand, and descriptive realism built around short-run notions of fixed cost and increasing returns, on the other. Third, this article contributes to the recognition of the important Italian contribution to economic theory in the early 20th century. Barone, Amoroso, Sraffa, and da Empoli were part of a common intellectual enterprise working in close interaction with key developments in Great Britain.

1. Introduction

The U-shaped average cost curve is one of the most widely used tools in microeconomic analysis. An introduction to the economic theory of the firm without presenting it, would – the world over – be considered either incomplete or eccentric. Its very ubiquity, however, tends to obscure the fact that the graph of the U-shaped average cost and the stories of returns to scale, competition and equilibrium that go with it are in fact part of a complex theoretical construct that emerged during the first three decades of the 20th century. Its complexity is due to a careful balance between (1) increasing returns and perfect competition and (2) short-run and long-run considerations in connection with the distinction between fixed and variable factors of production and (3) logical consistency and descriptive realism. And contrary to a popular misconception that careful balance is *not* the work of Alfred Marshall, but rather the work of his successors who tried – frequently in explicit opposition to him – to emerge from under his shadow.

The need to elaborate such a balance did, however, arise mainly from Marshall's original ambition to embrace comparative statics as well as economic progress through time in one single model in his *Principles*. The imprecision that such an all-encompassing theory would necessarily lead to was unacceptable to the methodological standards that economists were developing for themselves at the beginning of the last century. Subsequently, a more formalized theory of the

Correspondence may be addressed to Professor Jan Horst Keppler, CGEMP, University Paris-Dauphine, Place du Maréchal de Lattre de Tassigny, 75775 Paris cedex 16 and to Professor Jérôme Lallement, University Paris 5, MSE, GRESE, 106-112 boulevard de l'Hôpital, 75647 Paris cedex 13. We would like to thank Arrigo Opocher, University of Padova, Nicolas Chaigneau, University of Lille, as well as the participants of the Bari seminar on Attilio da Empoli for their comments, which have greatly helped to improve this paper. We would also like to thank the anonymous referees for their knowledgeable comments that helped to complete the present article. The authors remain, of course, solely responsible for any errors of commission or omission.

firm – of which the U-shaped average cost curve was to become the centre-piece – was developed in a painstaking process of discussion and refinement. This process involved some of the most eminent economic theorists of their time, among them, Enrico Barone, Francis Y. Edgeworth, Luigi Amoroso, Piero Sraffa, Arthur C. Pigou, Charles Schultz and Jacob Viner. It is noteworthy that there does not exist *one* first representation of a U-shaped average cost curve of a firm under competition, although Barone and Edgeworth need to be credited with important first steps. The concept really evolved step by step from a discussion that shows the difficulty of developing Marshall's intuitions into a framework solid enough to allow rigorous comparative statics analysis.

In this development of the modern theory of the firm, the role of Italian theorists, notably Enrico Barone and Piero Sraffa (before his emigration to England), both very much part of a national discussion on economic theory, deserves to be underlined. Their combined contribution, which was elaborated through a web of cross-references, amounts to an identifiable *Italian* contribution to the foundations of the modern theory of the firm. The historical contributions by the Italian theorists as well as by Edgeworth have been obscured by the, now classical, codification of the theory of the firm under competitive conditions by Viner ([1931] 1931) in Chicago. Viner's article on "Cost Curves and Supply Curves" synthesizes much of the earlier work done on the theory of the firm. Curiously, Viner mentions in passing Pigou, Sraffa, Shove, Harrod and Robertson but does not mention Barone and Edgeworth, the earliest developers of the U-shaped average cost curve. In the following years, the effervescence of the work on monopolistic competition in the two Cambridges monopolized interest. When the theory of the firm under competition regained interest after the war, the U-shaped average cost curve was, ironically, seen as a Marshallian concept. This misconception persists by and large until today.

This article thus provides a fuller account of the development of the U-shaped average cost curve than has been available to historians of economic thought so far. It also thoroughly qualifies the assertion that the U-shaped average cost curve is a Marshallian concept and shows that its development took place in opposition to Alfred Marshall, carefully combining different – partly contradictory – elements of the vast Marshallian legacy. In the process, the article will highlight the extent to which the concept of the U-shaped average cost curve still contains tensions between notions of short- and long-run equilibrium, pricing power and competition, realism and logical consistency. Far from wanting to de-legitimize the concept for teaching purposes, applied or theoretical research, the article wants to show its richness, but also its intricacies and limits, in all of which it is a perfect reflection of economic theory at large and which make for a far more interesting story than that which is usually told.

2. The Histories of an Idea

The concept of the U-shaped average cost curve did not spring fully armed out of the head of one single inventor. Its origins and history are interwoven with the complex history of interwar economic thought including the development of monopolistic competition theory. However, even given those complexities, the different references made to the origins of the U-shaped average cost curve even by informed observers are surprisingly wide off the mark.

Most commentators ascribe the origins of the U-shaped average cost curve to Alfred Marshall, usually with a reference to his work on decreasing and increasing returns. Several works in the history of economic thought (for example [Blaug \(1962\)](#), [Béraud and Faccarello \(2000\)](#), or [Ekelund and Hébert \(1975\)](#)) present their analysis of the Marshallian theory of supply on the basis of U-shaped average cost curves. This includes a marginal cost curve, at first decreasing than

increasing, eventually intersecting the U-shaped average cost curve at its lowest point. These works specify that these curves cannot be found as such in Marshall's *Principles of Economics* ([1890] 1920) and that their graphic representation is due to Jacob Viner's classic article "Cost Curves and Supply Curves".

There is no doubt that the origin of these U-shaped cost curves, both the average and the marginal one, is well prior to Viner, who wrote his article during the time when Attilio da Empoli was a Rockefeller Fellow in Chicago and published his *Theory of Economic Equilibrium* (1931). Closest in time to Viner are Henry Schultz his colleague at Chicago and Arthur Cecil Pigou, Marshall's successor in Cambridge. Schultz presents a U-shaped average cost curve in "Marginal Productivity and the General Pricing" (1929). In passing, makes a reference to the work of the French economist François Divisia, *Economique rationnelle* (1928) that contains a not further motivated U-shaped average cost curve (without marginal cost). Also in 1929, Pigou also adds to the appendix of the third edition of his *Economics of Welfare* a U-shaped cost curve. Sraffa, who arrived in Cambridge in 1926, had published "Sulle relazioni fra costo e quantità prodotta" that contains the U-shaped average cost curve one year earlier. On the basis of that article, Edgeworth, who read and wrote Italian fluently, had invited Sraffa to contribute an article to the *Economic Journal*, an invitation that gave birth to the famous article on "The Laws of Return under Competitive Conditions" (1926), a substantially altered version of the 1925 article.

Sraffa refers in his 1925 article to the fourth and final part of Edgeworth's series of articles throughout 1911 and 1913 in the *Economic Journal* that is called "Contributions to the Theory of Railway Rates". The fourth part of Edgeworth's long meditation on monopoly pricing contains the first known presentation of the U-shaped average cost curve as it is known today. He cites his (purely verbal) exposition of three lines in a review article in 1905 as a source for it, but he does not mention Barone, who in *Principi di economia politica* ([1908] 1936) provides an equivalent graphical representation of a U-shaped average cost curve. Barone does not plot quantity against average and marginal cost, as we would today, but against total cost curve, where the steepness of the lines drawn between various points on the curve and the origin indicate average cost and the tangents indicate marginal cost. Between Edgeworth and Barone the primacy for developing the U-shaped average curve needs to be shared in a somewhat unusual constellation.

3. The U-Shaped Cost Curve: A Subtle Compromise between Competing Notions

Before entering into the detailed discussion of the development of the U-shaped average cost curve, it is worthwhile to briefly recall its principal analytic features. This short review will allow grasping the depth but also the ambiguity of the concept, which explain its versatility as an analytic tool at the heart of microeconomics. The review will also highlight that this versatility comes at the price of blurring the boundary between short-run and long-run analysis (notably in form of the horizontal demand curve) as well as that between perfect competition and discretely sized firms.¹

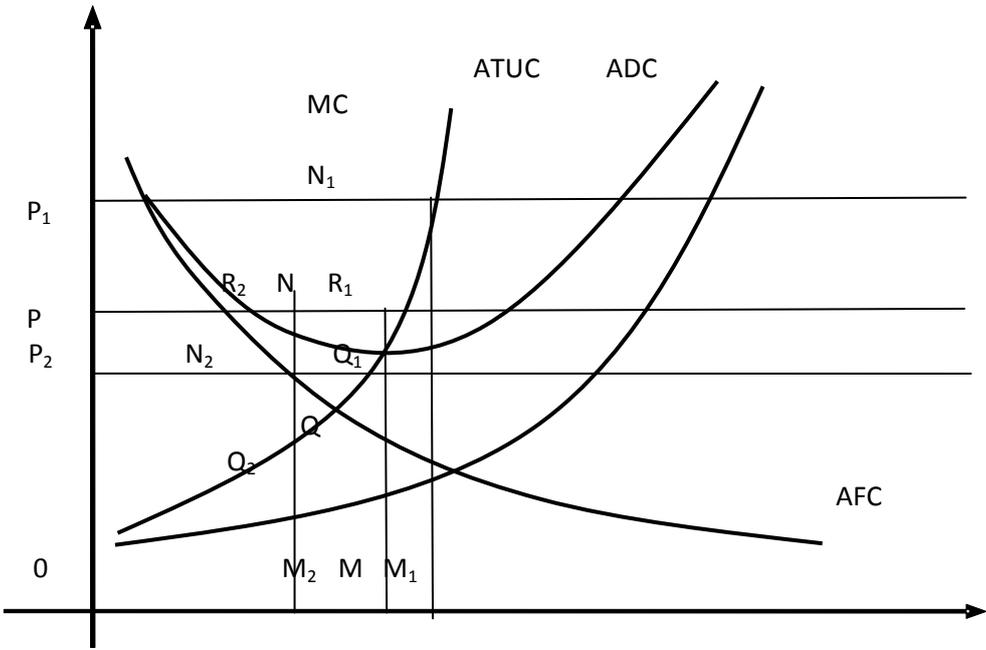
Today, the U-shaped average cost curve together with the marginal cost cutting it at its lowest point is considered the standard representation of the equilibrium of the individual enterprise under competitive conditions. The graph below follows Viner's canonical representation (Viner

¹ In a finite market, a discretely sized firm that produces under non-constant returns to scale will always possess some (even if small) degree of monopoly power in the short-run.

[1931] 1953, 202), in which ATUC stands for average (total unit) costs, MC for marginal cost, ADC for average direct costs (average variable costs) and AFC for average fixed costs per unit. Viner also postulates three short-run equilibriums for the individual enterprises in function of the three prices P , P_1 and P_2 . Viner's notion of the short-run assumes (implicitly) that prices move quicker than entry or exit can take place. Otherwise P_1 would attract entry of new firms and P_2 exit of existing firms. Either movement would drive price towards the only level that we would consider today consistent with equilibrium, price level P .

The U-shaped average cost curve is a rich construct that allows a number of different, not always entirely consistent, lines of economic reasoning to be developed. At the most basic level, it is a handy didactic tool to introduce undergraduate students to such fundamental economic issues as:

- Profit maximization
- Total, average and marginal cost
- Fixed and variable cost (as well as average variable cost and average fixed cost)
- Increasing, decreasing and constant returns to scale
- Competition,
- Elasticity of demand including horizontal demand curves
- Short-run supply curves derived from the marginal cost function
- Firm size and capacity, and
- Rent.



The neatness of this simple tool and the variety of empirical and theoretical notions that can be developed from it, guarantee appreciative audiences and a helpful introduction to a number of important microeconomic notions. However, treating such a wide range of issues comes at a cost. It is possible only because the U-shaped average cost curve combines a number of different economic features that sit uneasily together and that render it, in final instance, a fairly

ambivalent concept. The set-up chosen by Viner – one that is reproduced in countless textbooks – implies that the individual firm (1) sells its output at marginal cost and (2) possesses a discrete size due to a significant fixed cost. Today we would argue that selling at marginal cost implies perfect competition (absence of pricing power of the individual firm) and thus either atomicity of the individual establishment or production under constant returns to scale. Either condition is incompatible with a sizeable fixed cost.

There is, however, a trade-off involved that goes some way in explaining the complex and contradictory history of the origins of the U-shaped cost curve as well as its enduring attractiveness. While the coexistence of competition and discrete fixed costs leading to increasing returns over a significant range of the firm's production creates problems for logical consistency, it lends a significant amount of descriptive realism to the construct. By adding product differentiation, the theories of monopolistic competition would later enhance the logical consistency of the construct, maintain its status as a convincing analogy with observable economic reality but lose some of its analytical handiness. The U-shaped average cost curve instead presents the cost function of a firm operating in a competitive industry composed of firms producing identical goods and maintains thus the tension between elements indicating perfect competition (horizontal demand curve) and not so perfect competition (sizeable fixed costs, increasing returns to scale). It is significant in this context that the first graph of a U-shaped cost curve ([Edgeworth \(1913\)](#)) was intended to represent the cost function of a *monopolist* producing under *increasing returns to scale*, i.e. to the left of the nadir of the curve that is considered today to be the point of production.

From one point of view, the U-shaped average cost curve is a *short-run* equilibrium concept for the individual enterprise under perfect competition. All parameters other than output and variable cost are part of *ceteris paribus*, including – most importantly – the fixed cost, which can be taken as a proxy for the technology at the disposal of the different enterprises competing against each other. Only in the short-run, does the notion of a “fixed” cost make sense. The very definition of the long-run is that all factors of production can be adjusted.

However, from another point of view, the no-profit condition (due to perfect competition) in combination with a horizontal demand curve implies that a firm always fully exhausts its economies of scale. It thus arrives at a point where it finds itself in a situation “as if” it were operating under constant returns to scale (average cost equal to marginal cost), i.e. “as if” the fixed cost could always be precisely adjusted so that average cost would be minimized precisely at the level of the (exogenously given) price. Such malleability of the (no longer) fixed factor of production requires long-run analysis.

In the long-run, the notions of “increasing” and “decreasing” costs (the two sides of the “U”) cancel each other and vanish at the economically relevant point.² This observation was taken by Sraffa ([1925] 1975) as the basis for his claim that constant returns to scale should be considered

² The same thing happens to their constituent elements – fixed and variable costs. The transition to the long-run is precisely constituted by the fact that a short-run element (fixed costs) loses its economic relevance and is assumed to have effects, as if it was fully malleable (which it is only in the long-run). The only way to preserve the horizontal demand curve *and* independently determined fixed costs at the same time would be to assume market demands which are precise multiples of firm capacities – an assumption so absurd that it is rarely, if ever, invoked.

in theoretical economics as the only internally consistent hypothesis for the structure of production under competitive conditions.

The U-shaped average cost curve has the added inconvenience of requiring the conviction that the single firm's output is limited by cost of production rather than by demand, an assumption that has been challenged explicitly for the first time by [Sraffa \(1926\)](#). In the long-run, a horizontal demand curve to which firms equate their average and marginal costs under perfect competition ultimately renders the notion of an independently determined fixed cost (and with the notions of increasing returns to scale and of capacity) obsolete. The only way to justify the short-run notion of a discrete non-malleable fixed cost would be to abandon the assumption of perfect competition and to work with a "realistic" theory of monopolistic competition under product differentiation.³ "Reality", i.e. the snapshot of physical surroundings taken with naked, untutored eyes is, of course a short-run concept. In addition, it must necessarily focus on the unique, individual enterprise. In such a perspective, fixed costs, rent and capacity constraints are, of course, ubiquitous. In addition, all changes will be discrete and near-rationality inevitable.

As Sraffa points out, only if all factors of production (including those for which "fixed costs" are paid in the short run) were fully and instantly tradable, then a firm's size would be fully adjustable and constant returns with horizontal demand curves could indeed be considered the "normal" case. Of course, this is the long run rather than the short-run case – trading and producing capital goods takes time. However, maintaining in the same construction discrete fixed costs and increasing returns over at least some relevant range of production *and* a horizontal demand curve allowed, in the best Marshallian tradition a careful trade-off between (short-run) realism and (long-run) theoretical coherence. The most obvious weakness of the construct of the U-shaped average cost curve is also its greatest strength.

While the U-shaped average cost curve retains a central ambiguity, its development nevertheless managed to get rid of a great amount of "Marshallian baggage" (the expression is due to Joan Robinson with respect to Keynes' microeconomics) that were sowing confusion among the most eminent economists during the 1920s and the beginning of the 1930s. Despite this tension and perhaps because of it, the U-shaped average cost curve has firmly established itself at the heart of the economic theory of the firm as well as a useful introduction to the theory of the market. The way by which it arrived there is a long story and the topic of this article.

4. Early Precursors and the Marshallian Legacy

A full genealogy of the U curve would require us to go back far before Marshall to pay homage to the writers who worked on distinct elements of the theory of the firm before they coalesced in the form, in which we know it today. Without asserting exhaustiveness, we need to evoke two important 19th century sources – first Antoine A. Cournot, and then the group of analytical writers that worked without any explicit reference to the economic theory of the day, the "engineers".

³ This was, of course, the choice of many economists during the 1930s following the publication of Edward Chamberlin's *Theory of Monopolistic Competition* and Joan Robinson's *Economics of Imperfect Competition* in 1933. One should note though that this choice was possible only once the synthesis of firm behavior in form of the U-shaped average curve was available and had brought together the different elements of the discussion.

The theoretical formulation of the cost curves could have (but did not) already come from Cournot. In his analysis of monopoly, he formulated the all important equality between marginal cost and marginal revenue as a condition for profit maximization (Cournot, [1838] 1980, 45).⁴ Cournot also evokes the possibility that marginal cost be successively decreasing then rising (*ibid.*, 45). If he thus prefigures a U-shaped marginal cost curve, he never names marginal cost anything else but a “differential coefficient”, does not graphically represent it in a curve, and does not introduce average cost. In addition, when Cournot distinguishes between fixed cost and variable cost as factors of total cost, he does not introduce any dynamic considerations. This prevents integrating his emergent cost analysis into an analysis of market adjustment processes, where the Marshallian distinction of short and long term plays such a vital role.

Before becoming a central battleground of theoretical economics, the question of costs was also taken up from an empirical point of view by practitioners, in particular by mechanical engineers, who in the middle of the 19th century attempted to determine costs of production and tariffs for the use of railway. Charles Ellet in the United States, Alphonse Belpaire in Belgium, the engineer-economists Adolphe Jullien and Jules Dupuit in France or Dionysius Lardner in the United Kingdom, all undertook studies heavily relying on figures and statistics that calculated total costs and even average costs and cost variations, more or less similar to what we refer to today as marginal costs.⁵ However, these studies essentially focused on setting tariffs. Their authors never advanced toward formulating the cost functions explicitly or toward linking cost variations to variations of quantity in the context of an analysis of the laws of returns. Moreover, if we except Dionysius Lardner explicitly quoted by William S. Jevons and thus known to Marshall, there is no evidence that the central protagonists of our history of the development of the U-shaped cost curve knew about the works of the engineers and we can reasonably assume that their influence in subsequent development stayed marginal.

This leaves the work of Alfred Marshall as the proper starting point of the discussion. Marshall never used the concept of the U-shaped average cost in his writings, nor did he ever comment on either Barone ([1908] 1936) or Edgeworth (1913) in his writings. Nevertheless, it is futile to develop the history of the concept without presenting his writings on the theory of the firm. Marshall is not a precursor to the concept in an ordinary way. One could even argue that Marshall contributed to the development of the U-shaped average cost curve only in a *negative* way, by allowing several writers, notably Sraffa, to sharpen their tools in explicit opposition to

⁴ To simplify, we employ a modern expression that was not used by Cournot.

⁵ We can find a full report of these works in Ekelund (1971-1972), “Economic Empiricism in the Writings of Early Engineers”, as well as in Ekelund and Hébert (1999), *Secret Origins of Modern Microeconomics*. For example, Charles Ellet was trying to find out how to link the total expenditures of a railway company to different factors: number of passengers, tonnage of merchandise, distance traveled by locomotives, total length of the network (Ekelund (1971-1972, 181-188), Ekelund and Hébert (1999, Chapters 6 and 11). The key objective of his research was to verify whether the expenditure they calculated from their statistical data, tallied with the actual expenditure incurred by the railroad companies. Another example is the work of Alphonse Belpaire (Ekelund (1971-1972, 190) who aimed to calculate the costs of the various services the railway companies provided to their clients in order to allow them to set their tariffs correctly. Even if the statistical tables established as part of these empirical studies sometimes show, more or less deliberately, average cost and marginal cost next to total cost, it remains a fact that these costs are never interpreted as outcomes of cost *functions*.

him. In other words, Marshall's *Principles of Economics* constitute an unavoidable backdrop for many of the subsequent theoretical discussions.⁶

Much of the work towards the U-shaped average cost curve was developed, explicitly or implicitly, in response to the ambiguities of Marshall's economics. One might say that the U-shaped average cost curve expresses some of these ambiguities with more clarity than Marshall would have ever dared to express them in. Marshall's treatment of the combination of increasing and decreasing returns in the same establishment is exceedingly complex, mixing static with dynamic considerations as well as firm-level effects (internal economies) with market-level or economy-wide effects (external economies). The development of the U-shaped average cost curve goes hand in hand with the clarification of key economic concepts and a shift in methodological preferences: the success of the U-shaped average cost curve is also due to the fact that it presents itself as a concept of comparative statics.

It is well known that Marshall carefully elaborates on the theory of both, increasing and decreasing returns to scale. Following his elaboration of decreasing returns in connection with agricultural enterprises (Book IV, chapter 3), he considers industrial enterprises (i.e. those to which the U-shaped average cost curve was eventually intended to apply) to be characterized chiefly by increasing returns:

The chief advantages of production on a large scale are economy of skill, economy of machinery and economy of materials... (Marshall [1890] 1920, IV.11, 278)

These increasing returns (and here the difficulties start) were thought to apply in the short run as well as in the long run, or alternatively in a static as well as in dynamic sense.⁷ In the short run, average cost would decline with output due to the spreading of fixed cost over a greater quantity of output produced. In the long-run, increasing skills and dexterity (economies of learning) would exert further downward pressure on cost.

Marshall considers three factors that check the tendency towards monopolization that naturally results from these increasing returns to scale. None of them explicitly mentions decreasing technical returns to scale (i.e., capacity exhaustion), as would be the case with a U-shaped average cost curve. The first two factors mentioned by Marshall are product differentiation and the fear of spoiling the market (*ibid.*, 287). The third argument is the most important one – the decreasing organizing ability of the entrepreneur with respect both to the size of the establishment he runs *and* the lifetime of the enterprise.⁸ The principal factor that

⁶ For an exhaustive recognition of Marshall's unique and complex role at the heart of the development of neoclassical economic theory see Ekelund and Hébert (2002).

⁷ We do not distinguish here between pure static analysis, in which no economic variable other than price can be adjusted (the "time of the market" in Marshallian parlance), and the short run, in which variable cost and output but not fixed cost can vary.

⁸ The question to which extent this organizing ability defines the capacity of the company (its size, as well as its fixed cost, which can be taken as a proxy for the difficulty to reproduce the rare factor of "organizing capability" or "entrepreneurship") is taken up in the late 1930s by Nicholas Kaldor and Edward H. Chamberlin in their infamous exchanges of "monopolistic" vs. "imperfect" competition. Despite all the flaws of their bungled debate, the participants had by then at least clarified that they were exclusively talking about decreasing returns to scale with respect to the size of the enterprise. Comparative statics had won the day even for monopolistic competition theorists. Marshall's argument eventually found a rigorous elaboration in [Coase \(1937\)](#), who let the entrepreneur's organizing capacity

prevents the limitless expansion of the single industrial firm is thus a *dynamic* notion of decreasing returns of scale, i.e., the *decay* of the entrepreneur's organizing capacity over his own lifetime as well as over the lifetime of the company. The size of the individual firm is thus, in principle, not limited at any given point in time, but its limits become ever more palpable as it tries to exploit its economies of scale through time.

While Marshall prefers dynamic arguments to argue for limits to increasing returns in his explicit discussion of the subject, there are other passages in the *Principles*, in which he underlines the ubiquity of decreasing returns also in a static sense, including in industrial establishments. He states in the style that is all his own:

Excessive applications of any means to the attainment of any end are indeed sure to yield diminishing returns in every brand of business; and, one may say, in all the affairs of life. (*ibid*, V.8.4)

In the same sense, he sees the phenomenon of rent not confined to the scarce factor of land but as pertaining to any factor of production, in particular capital investments, which cannot be replicated immediately, i.e. which is in fixed supply at least for some amount of time:

That which is rightly regarded as interests on "free" or "floating" capital, or on new investments of capital, is more properly treated as a sort of rent – a quasi-rent – on old investments of capital... And thus even the rent of land is seen, not as a thing by itself, but as the leading species of a large genus. (*ibid*, V.8.6)

This seems a vindication of the short-run and of non-replicable factors with "fixed" cost providing some monopoly power (rent) at least in the short run.⁹

Rent, of course, is obtainable only under decreasing returns to scale, which amounts to saying that it is obtainable only if the factor in question is in limited supply. At this point, Marshall has already discussed the topic at length in Chapter 3 of book IV in connection with the decreasing ("diminishing") returns on the use of agricultural land. The chapter in question begins with the words:

The law of or statement of tendency to Diminishing Returns may be provisionally worded thus:

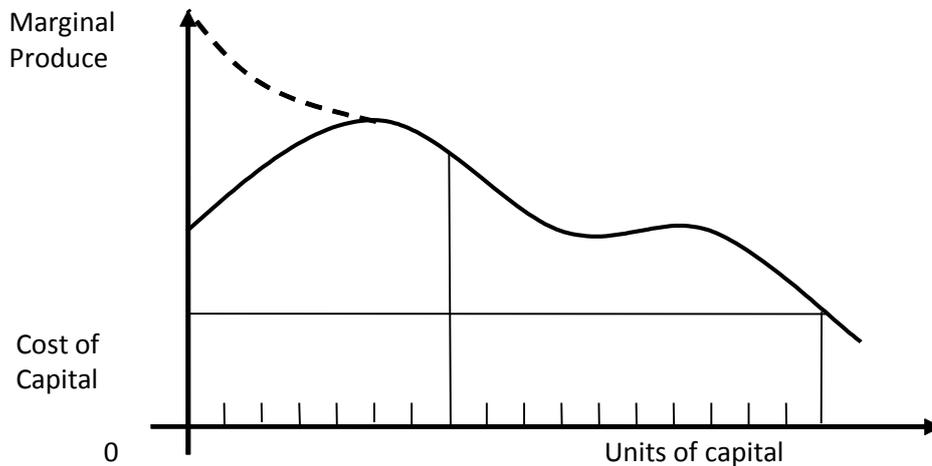
An increase in the capital of labour applied in the cultivation of land causes *in general* a less than proportionate increase in the amount of produce raised, unless it happens to coincide with a general improvement of agriculture. (*ibid.*, IV.3.1, 150)

Subsequent graphic examples show curves of marginal returns on subsequent units of capital outlays for cultivating a given piece of land (Figure 11 in footnote 2 on 155 and Figure 14 in footnote 1 on 158, see also the graph below). Unsurprisingly, returns decline over most of the

(in comparison with the transaction costs of the market) define the size of the individual firm in a framework of static maximization.

⁹ The attentive reader will notice how effortless Marshall weaves analogies of biological phenomena evolving in continuous time into his economic argument. The great theme of Alfred Marshall is, of course, the transition between different time horizons – all time horizons. One should therefore not be too surprised to read in the *Principles*, habitually considered the bible of demand-driven short-run economics, the very "Sraffian" statement that "... the value of a thing tends in the long run to correspond to its cost of production (Marshall [1890] 1920, V.3.7, 348)." This statement implies automatically a horizontal demand curve and *de facto* production under constant returns to scale.

range. What is rather surprising, however, is that the very first units of capital applied to a given plot of land show *increasing* returns, i.e., decreasing costs per unit of output. (The graph below is reproduced as found in the *Principles*, only the labeling has been changed to avoid Marshall's cumbersome notation).



Marshall, ever hedging his bets, indicates that the broken line (which would yield a continuously decreasing marginal produce) is the case for already cultivated land in an “old country” such as England, an assumption that he considers underlying Ricardo’s reasoning (*ibid.*, IV.3.2, 156). What is interesting for the discussion at hand is not so much Marshall’s ingenious reasoning drawn from the realities of agricultural life that motivates every portion of the curve, but the fact that his *Principles* already contain two graphs which combine increasing and decreasing returns – albeit in an usual system of coordinates and not in the context of an industrial firm.

The problem is that Marshall’s combination of different laws of returns operating in the same establishment is not subjected to any rigorous generalization. Marshall prefers to add endless qualifications to his observations – a habit that can be exasperating. In the context of the laws of returns, his comments next to a graph similar to the one above but with two peaks of even height (figure 15 in footnote 2 on page 159) are the following:

Of course, his [a farmer’s] return may diminish and then increase and then diminish again; and yet again increase when he is in a position to carry out some further extensive change as was represented by fog. 11. But more extreme circumstances, of the kind represented by fig. 15, are not very rare. (IV.iii.3, 159)

In an individual agricultural enterprise returns can be increasing, decreasing, increasing and decreasing again. Static and dynamic considerations are happily thrown together. It is small wonder that subsequent writers interested in heightening the scientific rigor economics developed such verve attacking Marshall’s microeconomics. However, in the bigger picture all these qualifications may not matter all that much, later in Book IV, Marshall states:

In other words, we say broadly that while the part which nature plays in production shows a tendency to diminishing returns, the part which man plays shows a tendency to increasing returns... If the actions of the laws of increasing and diminishing returns are balanced we

have the *law of constant returns*, and an increase produce is obtained by labour and sacrifice just in proportion. (*ibid.*, IV.13.2, 318)¹⁰

We may synthesize that one can find in the *Principles* separate elaborations of increasing returns in industry and of diminishing returns in agriculture. Both strands of reasoning contain *in nuce* their opposite. This intrinsic combination of opposite tendencies may under certain not specified conditions even arrive at a balance of the two countervailing forces – constant returns to scale. Marshall’s Cambridge pupils used to say “It is all in Marshall.” We may agree with them, however, we might be also tempted to add “yes, everything... as well as its contrary.”¹¹

Together with Sraffa (1926) it was Viner ([1931] 1953) who contributed most to the misunderstanding that the theory of the firm based on the U-shaped average cost curve was a synthesis of Marshallian economic thought. After chiding Marshall, rather surprisingly, for “terminological poverty” and for being “excessively simple”, Viner situates himself in a Marshallian tradition:

In recent years a number of English economists, notably Pigou, Sraffa, Shove, Harrod and Robertson, have presented in the *Economic Journal* a series of criticisms, elaborations, and refinements of the Marshallian analysis which, in my opinion, go a long way both towards bringing out clearly the contribution contained in its implications as well as in its explicit formulations, and towards completing and correcting it where that is necessary. (Viner ([1931] 1953), 199)

Viner’s criticisms are surprising to the extent that the problem with Marshall’s economics stem from his endless additions, qualifications and *ad hoc* examples expounded with great semantic inventiveness rather than excessive simplicity. One might add the fact that certain statements in the *Principles* that are at odds with each other are never fully reconciled. Anything goes. Increasing, decreasing and constant returns are possible at different stages of the enterprise lifecycle alone or in combination.¹² In particular, it is never fully explained how he

¹⁰ Marshall’s generalizations can be seductive but have to be treated with caution even in the context of his own reasoning. For instance, the entrepreneur’s organizing capacity responsible for decreasing returns (see above) is surly a “part that man plays”.

¹¹ Marshall’s tendency, and talent, to force conflicting opposites together is perhaps best displayed in his defense of the downward-sloping supply curve in a competitive industry in a framework of comparative statics. Mobilizing impacts internal to the industry but external to the firm to justify decreasing unit costs with increasing output – such as the emergence of a specialized workforce or the creation of a trade journal for better information – Marshall famously tried to combine increasing returns to scale and competition. Of course, he was once more flirting with irreversible, dynamic effects. Will labor skills or information be lost again once the industry shrinks? The disappointment with this treatment increasing returns under competition was a strong force behind the development of monopolistic competition theory that succeeded the development of the U-shaped average cost curve (see Blaug (1985) or Kepler (1994) for detailed discussions).

¹² For instance in Chapter XIII of Book IV of the *Principles* [1890] 1920, 314ff). Following the famous analogy between the representative firm in its industry and the trees in a forest Marshall concludes that the normal case “in those industries which are not engaged in raising raw produce an increase of labor and capital generally gives a return increased more than in proportion, [1890] 1920, 318)” is that of increasing returns. But when he represents normal supply, Marshall draws uniformly rising supply curves (for instance [1890] 1920, 344, 346, 384, 389, 391, etc). Curiously, the only supply curve in the *Principles*, which is not rising over the whole range, is used by Marshall to illustrate a case of (overall)

passes from the analysis of the firm in Book IV to the analysis of supply in Book V, which shows only systematically rising supply curves.

However, viewing Marshall solely through Viner's (or Sraffa's) lens, as is popular today, is equally misleading. As shown above, Marshall never envisioned any single comparative static concept to represent the individual firm such as the U-shaped cost curve. Below, we will show that also the development of the concept itself by Barone, Edgeworth and Sraffa owed only very little to an explicit discussion of Marshall's writings. The historians of thought illustrating Marshall's work with the U-shaped cost curve are not "wrong" in any definite sense; different elements of that curve can indeed be identified throughout the *Principles*. They only project a latter-day concept of comparative statics into a work whose ambition was primarily organized by concerns about (descriptive) realism and evolutionary change through time.

5. Enrico Barone: A First Fundamental Clarification

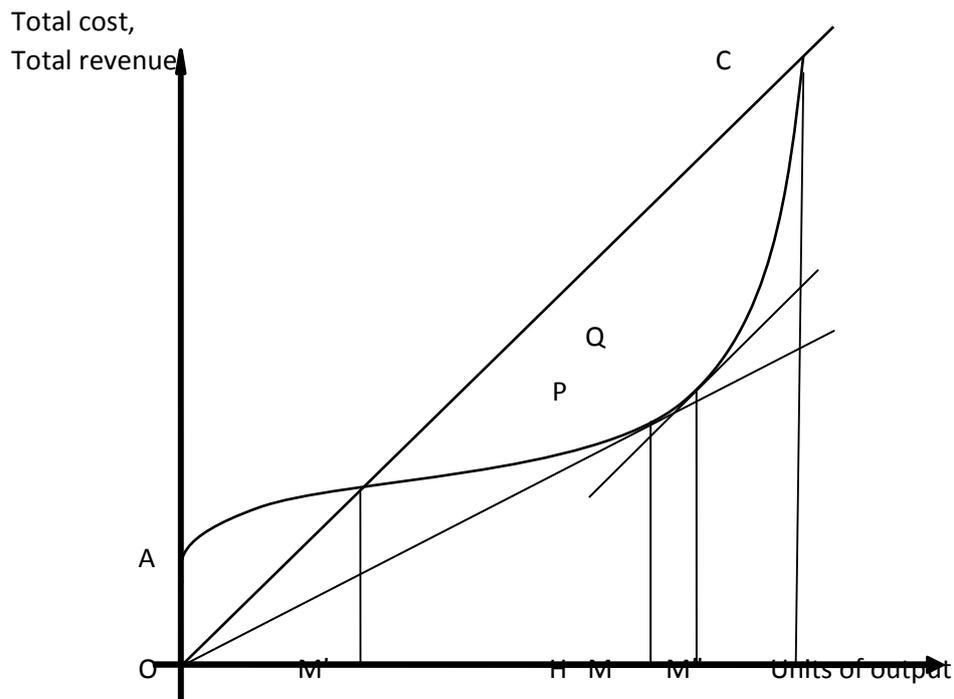
The first theorist who brought some coherence to Marshall's deep but unsystematic insights was the Italian Enrico Barone in his *Principi di economia politica*, first published in 1908. On a purely graphical level, it was Barone, who organized the firm's cost function in the now familiar diagram of quantities of output on the horizontal axis and cost on the vertical axis.¹³ On a conceptual level, Barone improves on Marshall in three important ways. First, he combines increasing and decreasing returns for the individual firm in a clear and unequivocal manner in a static framework. Second, he provides a clear statement of profit maximization, i.e. price equals marginal cost. Third he shows that competition will drive down prices such that profits will eventually equal average and profits will be zero.¹⁴

All this, Barone packs into one single graph. The history of the U-shaped marginal cost curve would have been quickly over, if Barone had not decided to display total cost and total revenue on the vertical axis rather than per unit cost and price. From the point of view of economic analysis, his insights are identical with those that can be derived from the U-shaped average cost curve as outlined in Chapter 3. The decisive Figure 12a on page 21 of the reprint edition of his *Principi di economia politica* shows the graph above that reproduces it without major changes.

increasing returns and it is first rising and then decreasing, which is precisely the opposite of the Viner's U-shaped cost curves.

¹³ Marshall never uses cost (price) and quantity of output for graphic illustrations of the laws of return, another reason to be careful using the apparatus of the U-shaped average cost curve to illustrate his work.

¹⁴ Barone [1908] 1936, Chapter 1 "L'equilibrio economico", 5-46.



Barone's comment on Figure 12a is the following. They are worth re-producing in some length because they allow in their very exhaustiveness some insights into the difficulties that even a major economist had making full sense out of his own construction.

Coming back to Figure 12a we see:

That in every firm there is a quantity OM that corresponds to maximum profit for the entrepreneur; and there are quantities M' and OM'' below and above which respectively, the firm would incur losses.

Point Q which corresponds to maximum profit is to the right of point P that indicates the limit between the zones of decreasing and increasing unit costs. [The footnote reads: It is to the right, because P is the point of tangency between the ray from the origin and the curve. Point Q of maximum profit has a tangency which is parallel to OC. And given that OC is steeper than OP and that the curve PC is such that from P to C the tangents become steeper, we can deduce that the point of maximum profit is to the right of P.]

The competition between entrepreneurs, decreasing the price OC, annuls profit and tends to push point Q towards point P, which means that it tends to force entrepreneurs to stay within the limits of decreasing costs, which they have surpassed to obtain maximum profits.

Competition will also define the capacity of the firm; this means that the [total] quantity produced will tend to distribute itself among the firms that produce at minimum cost such that each one of them produces OH which corresponds to the limit of decreasing costs. (Barone [1908] 1936, 24, our translation)

Before drawing the conclusions cited above, Barone had already motivated the shift from increasing to decreasing returns by a double argument. First, experience shows that decreasing costs do not continue forever (a modern expression would be "capacity exhaustion"). Second, the entrepreneur will not be able to increase certain factors of production (such as land) indefinitely, he will need to keep them fixed and thus hit decreasing returns (*ibid*, 21f).

In short, Enrico Barone reveals himself to be a true pioneer in the history of economic thought. While, he does not draw a U-shaped average cost curve himself, he certainly prepares its essential elements. His economic analysis goes far beyond the sparse comments that Edgeworth offers in his introduction of the first actual graph of a U-shaped average cost curve (Edgeworth (1913)). In fact, only Sraffa ([1925] 1975) was the first to fully grasp all the implications of Barone's construction. Among the authors that Barone is drawing on, Marshall is mentioned but only Cournot, Walras and Pareto are recognized "as the great masters of *synthetic economics*" to which he pays tribute.

In the introduction of his work, Barone excuses himself having simplified and generalized in a book that was primarily intended as a textbook for business students. Addressing himself, in particular, to the readers of the *Giornale degli economisti*, he apologizes for using graphics (and later static analysis) due to the didactic requirements of teaching. Today's textbooks do not come with apologies to the readers of the *American Economic Review*. Barone's comments also show that he was aware of the advance that he had taken over his prospective readers.

6. Francis Y. Edgeworth: The First U-shaped Graph of an Average Cost Curve

Edgeworth's "Contributions to the Theory of Railway Rates – IV" (1913) has the subtitle "Digression on Professor Pigou's Theories". Rather than commenting only on Pigou's 1908 opus *Wealth and Welfare*, notably the chapters on monopoly and railway rates, Edgeworth engages in a wide-ranging discussion of issues relating to demand and supply under different laws of returns that not only involves Pigou and the inevitable Alfred Marshall but also Pareto, Hobson, Auspitz and Lieben, Ripley, Ashley, Taussig as well as Gide and Rist. The main objective of the article is to sharpen the notion of equilibrium of monopoly under different laws of returns. Edgeworth, in particular, tries to clearly distinguish marginal and average costs an attempt that is marred by the construction – in a desire of symmetry -- of a "marginal demand price" that seems to overlook that demand is by itself an expression of marginal (an not average) utility (see Edgeworth (1913), 211). The U-shaped average cost curve is introduced in this context in a strange mixture of off-handedness and gravity. The construction does not hold an important place in the general structure of the article's argument. It is introduced with the words:

So far we have supposed the curve of marginal supply prices to be ascending. Now let us consider a descending [sic] curve of the sort such as SB in Fig. 3... [see below].

In the following, Edgeworth limits himself to pointing out that any point on decreasing returns to scale portion of the U-shaped average cost curve will not yield a stable equilibrium under competition (but might do so under monopoly). At the same time, Edgeworth dedicates a full page to the graph and its careful mathematical exposition. That page – written in a different font – however, remains largely unconnected to the rest of the article. The only hint that Edgeworth provides with respect to its *economic* significance is a reference to his review of Cunyngame's *Geometrical Political Economy* in the *Economic Journal* (1905). In this review, which is concerned with the transition of an individual firm's cost curves to (market) supply curves under competition, he states:

Keeping to the *régime* of competition we may illustrate the successive cost curves by parallel lines positively inclined to the axis x . First let the height, or distance from the origin in an upward direction, of a successive cost curve be greater the larger the scale of total

The fact that Edgeworth's meanderings actually produced the first U-shaped cost curve could almost be dismissed as a coincidence, a fluke, if he had not added to it an extensive mathematical footnote that contains all the relevant developments known today, *without* any distinction between firm supply and market supply. Intuitively, the modern reader would read x as being individual supply. Given the presence of non-horizontal demand curves, however, one needs to see Edgeworth's construction as the cost (supply) curve of an individual monopolist rather than as the cost curve of a firm under competition.

Compared to Barone's reasoning in terms of total cost, the advantage of representing the cost function in terms of unit costs instead of in terms of total cost is evident: the graph of the U-shaped average cost curve actually declines throughout the domain of decreasing costs, is horizontal where returns are constant and rises again throughout the domain of increasing costs. The position of the graph of the U-shaped average at the heart of the theory of the firm comes also from this powerful correspondence between graphic and verbal representation. The mathematical genius of Edgeworth is in full display in the lengthy mathematical note accompanying the graph reproduced above. Defining $AB = b$, $OA = 2a$ and $OS = 2a^2 + b$, Edgeworth defines the marginal cost curve as

$$Y = b + \frac{1}{2} (x - 2a)^2 .$$

Defining the marginal cost curve this way (very high at the beginning, strongly decreasing after) Edgeworth obliterates the necessity of introducing any "fixed cost" separate from marginal (variable) cost. This is an important point for Edgeworth who considers it "well to remind the reader that there is something arbitrary... in the distinction between "prime" [variable] and "general" [fixed] costs (Edgeworth (1913), 209)". Integrating this marginal cost function, he obtains in a second step the total cost curve, which he calls

$$xy = \int_0^x Y dx = bx + \frac{1}{6}x^3 - ax^2 + 2a^2x .$$

The average cost curve is then given as

$$y = b + 2a^2 - ax + \frac{1}{6}x^2 .$$

Edgeworth also clearly defines the point of cost minimization (albeit without elaborating on its economic significance):

There is a minimum of y at the point of intersection S' between the two curves. This property is general; since $Y = dx/dx = y + x (dy/dx)$; and accordingly when $Y = y$, $dy/dx = 0$. (Edgeworth (1913), 214)

Clearly, we have here the essential elements of the analytic apparatus surrounding the U-shaped average cost curve, the only thing that is missing (and that Edgeworth could have easily found in Barone) is the statement that *price* will be equal to marginal cost and average cost in equilibrium.

One can only speculate why the impact of Edgeworth's exposition of the U-shaped average cost curve was confined to a footnote, twelve years later, in a review article of a yet unknown Italian economist by the name of Piero Sraffa. The first reason is probably that Edgeworth did not recognize (or if he did, he did not state it) that only competition would push an entrepreneur to the cost minimizing point where average equals marginal cost.

Throughout the article, Edgeworth is consumed by the symmetry between the demand and the supply apparatus. Consequently, he follows Pigou in paying great attention to a "curve of marginal demand prices", the line DD_2 in the graph above, which traces average rather than

marginal utility (see Edgeworth (1913), 211).¹⁵ This concept is primarily relevant for a monopolist able to perfectly discriminate his customers. Another reason is that Edgeworth's capacity of obfuscation and imprecision rivals that of Marshall. Immediately following the rapid, precise and important mathematical analysis of the interaction of the two cost curves, the reader is treated to an eccentric, ephemeral and ultimately false justification of the precise dimensions of the graph above. Edgeworth writes:

In the figure the unit a [defined as one half of the production needed to minimize marginal cost] is taken as three quarters of an inch, and b [defined as the height of marginal cost at its minimum] is taken to be half an inch. Accordingly, $OS (= M'Q') = 2$ inches. (Edgeworth (1913), 214)

This is mathematically impossible. OS which correspond to $Y(0)$ or marginal cost with zero production is defined as

$$Y = b + \frac{1}{2} (x - 2a)^2 \text{ which for } x = 0 \text{ will yield } Y = b + 2a^2 .$$

Substituting $\frac{3}{4}$ and $\frac{1}{2}$ for a and b will yield $Y = \frac{1}{2} + 18/16 = 13/8 \neq 2$. Most likely Edgeworth just misread his own earlier equation as $Y = b + 2a$. Thus shrouded in arcane discussions about the "curve of marginal demand prices", marred by several errors, Edgeworth's exposition leaves today's reader somewhat unsatisfied. Perhaps the contemporary readers' reaction was not much different, which would explain why the first appearance of a U-shaped average cost curve was passed over almost with indifference, with the significant exception of two Italian writers.

One of them is Luigi Amoroso, who echoes Edgeworth's algebraic representation eight years later in his *Lezioni di economia matematica* (1921) with a linear marginal cost function. Defining total cost Θ for output x by the equation

$$\Theta (x) = 9 + 2x + x^2,$$

He subsequently derives average (unit) cost as

$$u = (9 + 2x + x^2)/x \quad \text{which yield a U-shaped curve and marginal cost as}$$

$$m = 2 + 2x.$$

He also solves for $x = 3$ by setting $u = m$ showing with the help of numerical tables that an output of $x = 3$ will yield minimal average cost (Amoroso (1921), 171). Amoroso, introduces these

¹⁵ Today, we would call the concept an "all-or-nothing" demand curve (Layard and Walters 1978, 150). It refers to a situation, in which a monopolist is able to appropriate the *whole* (consumer) surplus through perfect discrimination. Edgeworth's exposition of the concept is complicated by the fact that his monopolist has a strongly rising *marginal* cost curve. In the discussion of the "curve of marginal demand prices" Edgeworth even commits two (closely related) errors. In the first instance, he stipulates that a perfectly discriminating monopolist would produce at the point where the "curve of marginal demand prices" intersects the supply (marginal cost) curve (Edgeworth 1913, 211). In the second instance, he states that in the case of a downward-sloping marginal cost curve, a monopolist attempting to maximize the sum of consumer and producer surplus, a "monarch of enlightened benevolence", would again equate the "curve of marginal demand prices" DD_2 to SS_2 , i.e. the marginal cost curve, in the graph above (Edgeworth 1913, 215). Both times, he is wrong. The maximization of the sum of producer and consumer surplus implies equating marginal (rather than average) utility and hence the ordinary demand curve with marginal cost. Once the surplus is maximized, perfect discrimination can certainly affect its distribution but not its size.

equations with the same detachment, with which he has provided specified equations for much of economic theory in the rest of his book, albeit without providing any references.¹⁶

Anyway, Amoroso puts the emphasis on his concept of “virtual cost” (*costo virtuale*) which he defines as

$$v = u \quad \text{if} \quad u \geq m \text{ or}$$

$$v = m \quad \text{if} \quad u < m.$$

For Amoroso, the virtual cost defines the economically relevant cost that is equated to price by a profit-maximizing entrepreneur. The reasoning is that an entrepreneur will not operate when he would be making losses equating marginal cost to price and hence insist on equating price with marginal cost. Once $m > u$, marginal cost will become the reference to maximize profits.

The designation “virtual cost” (as opposed to “real cost”) could be construed as implying that notions of cost outside the equilibrium point where average cost equals marginal cost is not economically relevant. This, however would be mistaken, as for Amoroso, the lowest point of the average cost curve (the “punto di fuga” as he calls it) is not necessarily the point of production. Individual entrepreneurs, more efficient than their competitors might move beyond that point and produce where marginal cost is larger than average cost – without that price adapts to a lower equilibrium.¹⁷ Unsurprisingly, the concept of “virtual cost” was important mainly for writers interested in disequilibrium economics such as Attilio da Empoli (see below).

Equilibrium theorists such as Sraffa and, in his wake, Pigou took Edgeworth’s work (including the characteristic U-shaped *marginal* cost curves) as a reference. However, without a convincing economic explanation, marred by two errors surrounding its exposition, Edgeworth’s contribution did not find the echo that it would have deserved. It remained left to Sraffa and the subsequent writers leading up to Viner to establish the U-shaped average cost curve as the central concept of microeconomics that it is today.

7. Piero Sraffa – Refinement and Rejection of the U-shaped Average Cost Curve

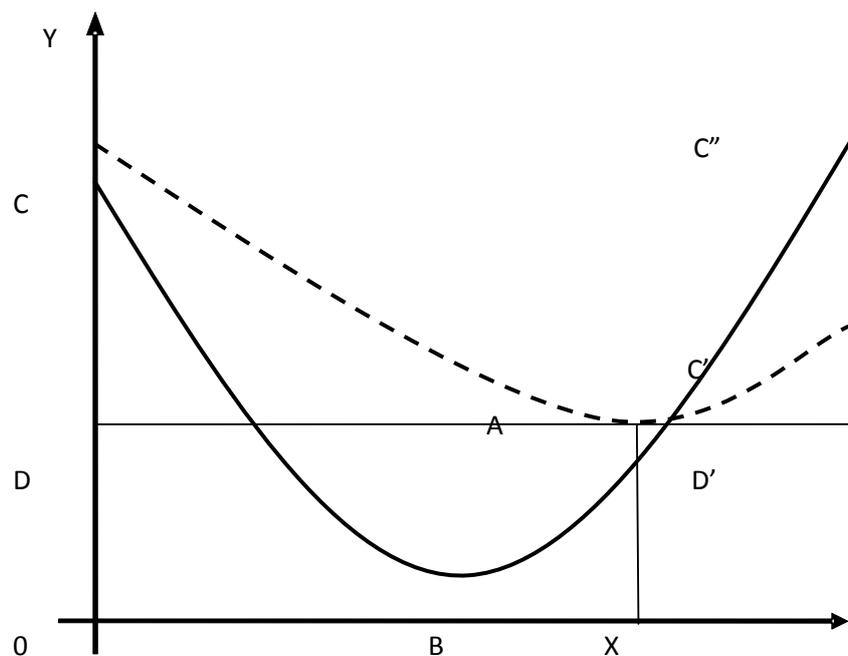
Piero Sraffa plays a uniquely ambiguous role in the further development of the concept of the U-shaped average cost curve. On the one hand, it is Sraffa who re-presents, with explicit reference to Edgeworth, the concept in his 1925 article “Sulle relazioni fra costo e quantità prodotta”. In addition, Sraffa makes an important adjustment: he now presents the U-shaped average cost

¹⁶ In his introduction, he mentions among the writers that he feels indebted to: Gossen, Walras, Pareto and Pantaleoni but neither Barone nor Edgeworth.

¹⁷ Amoroso has no difficulty with the notion that entrepreneurs might earn positive rents while prices stay stable as under perfect competition. As late as 1938, in his *Principii di economia corporativa* he would provide multicolored graphs depicting “differential rent” for differently efficient firms operating under the same horizontal demand curve (Amoroso 1938, 152). Today we would argue that new competitors will catch up with the most efficient firm and push down prices to the point where they equal average cost thus constituting the only possible point of stable equilibrium. Alternatively, the most efficient firm would have some monopoly power and hence be working under a downward-sloping demand curve. Sraffa ([1925] 1975) cites Pantaleoni (1923) making the same point as Amoroso and rejects this reasoning on the basis of the point made above: any divergence from the point where marginal costs equals average cost could not possibly constitute equilibrium. (Sraffa [1925] 1975, p. 34, FN 16)

curve as the representation of a firm *in competition*, including a horizontal demand curve. By and large, Sraffa presents and discusses the resulting graph in a manner that is fully consistent with today's textbook expositions. However, Sraffa does not only undertake a crucial refinement of the concept, he also repudiates it – not once, but twice.

The first time he does so is in the latter part of the 1925 article arguing that constant returns to scale is, in fact, the only hypothesis fully consistent with perfect competition. The second time he does so is in his famous article of 1926 “The Laws of Return under Competitive Conditions”, in which he argues that the only hypothesis consistent with a production function with non-convexities (such as increasing returns to scale) is monopolistic competition theory, of which his article became an important building block. In the middle, of the two arguments stands the firm with a U-shaped average cost curve operating under competition. The graph Sraffa produced was the following:



The ambiguity of Sraffa's position in this context results from the fact that he repudiates a concept that he himself has contributed creating. No one had ever before associated a U-shaped average cost curve with perfect competition. The only precedent is, of course, Barone's total cost wave with identical analytic properties. Several quotes attest that Sraffa had carefully read the first chapter of Enrico Barone's *Principi di economia politica* ([1908] 1936), although he does not cite him directly in connection with the U-shaped cost curve but only in connection with his theory of rent (Sraffa ([1925] 1975), 21-22). To a large extent, Sraffa grafts Barone's reasoning on Edgeworth's graph and adds a horizontal demand curve that corresponds to Barone's price line tangent to the total cost curve. In doing so, he refers to Edgeworth, but not to Barone. In addition, Sraffa is at the source of the misunderstanding that the U-shaped average cost curve is a Marshallian concept, as he employs it in order to synthesize *his* reading of Marshall (Sraffa ([1925] 1975), 29f).

Sraffa's insights, despite their great originality were not developed in isolation. The footnotes of his work show that he had read Barone closely. Italy at the end of the 19th and the beginning of

the 20th century was also a fertile breeding ground for economic reflection, where theories of partial and general equilibrium are hotly debated (see, for example, Gallegati (1990)). Sraffa, in particular, was nominated at the University of Perugia in November 1923 and put in charge of the course on general political economy. The preparation of that course, had forced Sraffa to occupy himself with the dominant academic theory, i.e. Marshallian marginalism (see Roncaglia (1990) and Naldi (1998)). The 1925 article is the result of Sraffa's research for his lectures and it were only his lectures that pushed him to pose the question of the relation between cost and quantity produces, or in other words, the question of returns (Lallement (2004)).

There are two ironies hidden in this process. First, Sraffa constructs a graph together with the appropriate economic reasoning of a firm with increasing returns to scale operating in a competitive, only to better repudiate it, little later. In other words, he designs a straw-man to better tear him down. Second, and this is perhaps the stronger irony, the straw-man survives in order to go on and become the generally accepted synthesis of the Marshallian theory of the firm as well as a fundamental building block of modern microeconomics.¹⁸

Let us examine first the form of the supply curve of a single representative firm. Let us trace on the abscissa (see figure 2 [above]) the quantities of the commodities produced by this enterprise, and on the ordinate the corresponding units costs, i.e. the total cost associated with each quantity divided by the number of units produced.

In order to satisfy the conditions mentioned above [the existence of static equilibrium under competition], this curve needs necessarily to be of a well specified type. First, it could not display increasing costs over its whole range: because in this case competition would tend to render the size of each firm infinitely small and increase their number indefinitely, and, due to the need of each one to reduce production in order to reduce costs, there would be no possibility to reach any equilibrium. The curve needs thus to display in all cases initially decreasing costs. Second, it could not do so entirely, because in this case, a firm must acquire the monopoly in its industry, which contradicts the hypothesis of competition. The supply curve of the representative firm will thus have in every case a shape of type CC'. (Sraffa [1975] 1975, 31f)

This is the U-shaped average cost curve as we know it today. The story of Piero Sraffa and the U-shaped average cost curve, however, does not end here. Having set the U-shaped average cost curve over a horizontal demand curve in an ensemble that will henceforth serve economists to explain the theory of the firm, Piero Sraffa, immediately proceeds to dismantle it. And he will not dismantle it once, but twice. His first critique takes place in the chapters of his 1925 article immediately following the presentation of the graph of the U-shaped average cost curve, in which he deduces that full tradability of factors over all industry would make the equilibrium point the only meaningful point of production.

If one assumes that all factors of production are used by a great number of industries (and hence that they are also perfectly transferable from one to the other), their remuneration, from the point of view of each industry is fixed and cannot be considered, from this particular point of view as a rent. (Sraffa [1925] 1975, 33)

¹⁸ The remaining chapters will show that the concept was transmitted through Sraffa and Pigou.

The idea of a “fixed factor” of production (and with it the notion of increasing returns) becomes thus meaningless and constant returns to scale the only tenable assumption for the production function (decreasing returns have already been rejected due to the fact that existing firms can always be replicated).¹⁹ This point is also underlined by the fact that the *marginal* cost curve is also U-shaped, just as with Edgeworth. Increasing returns are not due to the progressive amortization of some fixed cost but to a genuine and concomitant decline in the marginal cost (an increase in the productivity) of *all* factors. (In fact, the vision of Edgeworth, Sraffa and later Pigou would work perfectly well in a one-factor model.) With such malleability of factors (there are no “fixed factors” of discrete size) firms are free to adjust their production at the most advantageous point:

The marginal cost curve will cut the average cost curve each time at the point of maximum efficiency (A), point that also constitutes the only equilibrium possible. (Sraffa [1925] 1975, 33)

However, given the perfect malleability of factors there is little reason that marginal costs should now continue to rise faster than average costs:

In the perfectly possible case, in which for some (or even for all) quantities of product, individual marginal cost is constant, the marginal cost curve will coincide with the average cost curve over the interval corresponding at those quantities and in these limits, the equilibrium will be undetermined, given the definition of competition, that we have followed until now. (*ibid.*, 33)

The stage is set for the final conclusion that constant returns to scale constitute the only theoretically admissible assumption for the cost function in competitive industries.²⁰

The second attack that Sraffa rides against the U-shaped average cost curve comes one year later in his article on the “Laws of Return” (1926) and his famous “turn towards monopoly”.²¹ The *Economic Journal* article of 1926 had been written at the request of Edgeworth, who knew the 1925 article and subsequently organized the inter-cultural transfer.²² Synthesizing and sharpening

¹⁹ The idea that one (and only one) factor of production should be indivisible and fixed (at least in the short-run) whereas all other factors of production as well as the production function itself should be fully differentiable is indeed, at first sight, a logical inconsistency. The fact that the factor in question is often equated with “capital” surely the most malleable of all factors of production only heightens that inconsistency. However, the notion of indivisibility and hence production under increasing returns to scale can be justified if the fixed factor in question is identified with the specific informational content of a firm, its specific know-how, its brand, its patents. Given that the fixed factor thus established the uniqueness of a firm it also becomes non-tradable as a factor of production separate from its firm.

²⁰ From this point of view [the insignificance of either rising factor costs or external economies for individual industries], which constitutes only a first approximation to reality, we must thus admit that commodities are generally produced under constant returns. (Sraffa [1925] 1975, p. 49)

²¹ It is necessary, therefore, to abandon the path of free competition and turn in the opposite direction, namely, towards monopoly. (Sraffa 1926, 542)

²² The 1926 article has a complex and perplexing structure. In part, synthesis of the 1925 article, in part synthesis of monopolistic competition theory, Sraffa uses it also to respond to the complaints of Clapham’s article “Of Empty Economic Boxes” (1922). Clapham, Professor of economic history, underlines that economic theory provides the conceptual categories but does never provide the empirical content which is supposed to fill them. The theoretical categories thus remain “empty boxes”. Pigou replies in 1922 that one should not despair about filling the empty boxes one day.

the results of his earlier article, he refuses once more the, today commonplace, idea that *de facto* constant returns to scale could result from the co-existence of the competing forces of increasing and decreasing returns in the same establishment – as is the case with the U-shaped average cost curve.

The absence of causes which tend to cause the cost either to increase or to diminish appears to be the most obvious and plausible way from which constant costs can arise. But as these constitute the most dangerous enemy of the symmetry between demand and supply, those writers who accept this doctrine, in order to be able to relegate the constant costs to the category of theoretical limiting cases which in reality cannot exist, have persuaded themselves that they are something extremely complicated and improbable, since they “can only result from the accidental balancing of two opposite tendencies; the tendency to diminution of cost... and the tendency to increase of cost” (Sidgwick, *Principles of Political Economy*, 1st ed., 207; to the same effect see Marshall, *Principles*, IV.XIII,2, and *Palgrave’s Dictionary*, *sub voce* Law of Constant Return). (Sraffa (1926), 541)

In one of the most celebrated turnarounds in the history of economic thought, one page later Sraffa joins the chorus and insists on increasing returns to scale as the empirically most likely condition for a firm. In order to understand this seeming contradiction, one needs to see that Sraffa rejects the notion of a U-shaped cost curve, i.e. the notion of a firm of a determinate size working under perfect competition, once on theoretical and once on empirical grounds. On theoretical grounds, he rejects it as a concept for the long-run (in which increasing returns prevail, but are indistinguishable from general technical progress) and once as a short-run concept (in which decreasing returns prevail, but can be overcome by adjustments) (Sraffa (1926), 538-540). Concerning empirical observation instead, matters are different and he states:

Everyday experience shows that a very large number of undertakings – and the majority of those which produce manufactured consumer goods – work under conditions of individual diminishing costs. Almost any producer of such goods, if he could rely upon the market in which he sells his products being prepared to take any quantity of them from him at the current price... would extend his market enormously. (Sraffa (1926), 538-540)

We know from Sraffa’s further work that he would subsequently stick with the first rather than with the second line of reasoning. Sraffa’s preference for “objective” economic categories, unequivocally measurable in number, weight or size would instinctively draw him to constant returns to scale and fixed coefficients of production. His insistence on logical consistency would be a direct corollary.²³ Monopolistic competition with its differentiated competitors each one working with an indivisible (and hence not measurable) fixed factor of production could not attract him in the same manner in the long-run.²⁴ What is remarkable, however, is that both

Sraffa, however, argues that one has to understand that the question of returns is not, contrary to Clapham and Pigou, an empirical question. Where Clapham and Pigou, in the continuity with Marshallian analysis, placed the debate in the domain of realism, Sraffa goes one step back and places it in the realm of logical coherence.

²³ We are indebted to Arrigo Opocher for raising this point.

²⁴ It is a proof of the remarkable fertility of Sraffa’s economic insights that even a sketch of empirical observations (that he quickly abandoned as far as concerned his own work) was able to crystallize the scattered observations of different writers into the new field of monopolistic competition theory. The key step was, of course, that Sraffa clearly formulated the idea (that Marshall had hinted at earlier) that

concepts – constant returns to scale and monopolistic competition – are united in their opposition to the U-shaped average cost curve. What is most paradoxical in this story, however, is that that one of the fiercest – and most successful – critics of Marshall’s writings also gave Marshallian economics its most popular and enduring representation by synthesizing it in the graph of the U-shaped average cost curve.

8. Jacob Viner – An Endpoint without Conclusion

In the coming years, the U-shaped average cost curve becomes step-by-step the central microeconomic concept as which we know it today, and this despite Sraffa’s double salvo against his own construction. Pigou adds the U-shaped cost curves to the 3rd edition of *The Economics of Welfare* (1929) freely drawing on his article “An Analysis of Supply” published a year earlier in *The Economic Journal*. Pigou represents precisely the graph of Sraffa’s 1925 article “Sulle relazioni fra costo e quantità prodotta” and we may reasonably conjecture that he knew that article (as he exactly reproduces not only the shape of the lines but also the distinctive interruptions of the average cost curve (see above).

Pigou is also an important interloper between European and American economists. We have already mentioned that Jacob Viner’s article “Cost Curves and Supply Curves” ([1931] 1953) can be considered the definite representation of the U-shaped average cost curve. While it is well possible that Viner never read Sraffa, Edgeworth or Barone, he does make reference to Pigou, who thus constitutes the link between the European and the American development of the U-shaped average cost curve.

Another potential source for Jacob Viner is Henry Schultz, who is posted at the University of Chicago since 1926, just like Jacob Viner. His article of 1929, “Marginal Productivity and the General Pricing” contains a graph of the U-shaped average cost curve for an individual enterprise (Schultz (1929), 532). Schultz does not mention his source and does not accord it any special mention, as if the use of U-shaped cost curves was by now established usage. Schultz does make reference in his article to the *Economique rationnelle* by [François Divisia \(1928\)](#), which contains industry supply curves that have been obtained by horizontally adding individual supply curves ([Divisia \(1928\)](#), 135f). These supply curves of individual enterprises correspond to U-shaped cost curves if one takes into account the fact that Divisia, following the tradition established by Cournot and Walras, develops his curves in a diagram, where he places quantity on the vertical and price on the horizontal axis.

Viner, however, cites neither Divisia nor Schultz and insists on having been the first one to use U-shaped average cost curves – a claim, which we know by now is rather spurious. He does mention briefly Marshall, Pigou Sraffa, Shove, Harrod and Robertson and states:

The indebtedness of the present paper to their writings is considerable and is freely acknowledged. (Viner ([1931] 1953), 199)

the individual entrepreneur producing under increasing returns to scale would face a downward-sloping demand curve due to product differentiation. At the same time, Sraffa’s lack of enthusiasm for the strand of theory he had been instrumental in creating can also be understood: from today’s point of view monopolistic competition theory appears more an attempt to formalize than an attempt to supplant the *ad hoc* arguments of Marshallian economic thought such as the entry and exit of firms or product differentiation.

Only to continue:

But I have been presenting charts such as those contained in this article to my students at the University of Chicago for a long period antedating the writings referred to above, and if in the course of years these charts have undergone substantial revisions and, as I am convinced, correction, chief credit is due to the penetrating criticisms of my students. (Viner ([1931] 1953), 199)

By crediting a whole series of fellow economists as well as his students in general terms, Viner frees himself from the obligation to attribute more precise scholarly credits. While claims of the isolation and independence of American economics in the interwar period have been made elsewhere (see [Chamberlin \(1961\)](#)), the precedence of the development of an important new concept is habitually decided on the basis of publication. Concerning the famous long-run envelope of the short-run average cost curves (see below), Viner's claim to primacy is on much more solid ground, although the lack of any references to the work of Attilio da Empoli that evolved at the same time in close geographical and intellectual proximity is also unsatisfying.²⁵

While Viner's treatment of his sources remains unconvincing, he is rightly credited for establishing the definite version of the U-shaped cost curve and significantly advancing its discussion. Viner's great contribution was the clear distinction between the short and the long run and the concomitant distinction between fixed and variable factors of production.

The short-run is taken to be a period which is long enough to permit of any desired change of output technologically possible without altering the scale of the plant, but which is not long enough to permit of any adjustments of scale of plant. It will be arbitrarily assumed that all of the factors can for the short-run be sharply classified into two group, those which are necessarily fixed in amount, and those which are freely variable. (Viner ([1931] 1953), 202)

This distinction is reiterated in the graph of the U-shaped cost curve itself (see also Chapter 3). Contrary, to the graphs of Barone, Edgeworth, Sraffa and Pigou, Viner's graph draws a monotonously rising marginal cost curve. This implies that the marginal cost curve contains only variable cost. The former writers had drawn U-shaped marginal cost curves (or the equivalent of it, in the case of Barone) which implies an amalgamation of all factors of production.²⁶

Viner also provides helpful definitions for what an economic "long-run" really implies:

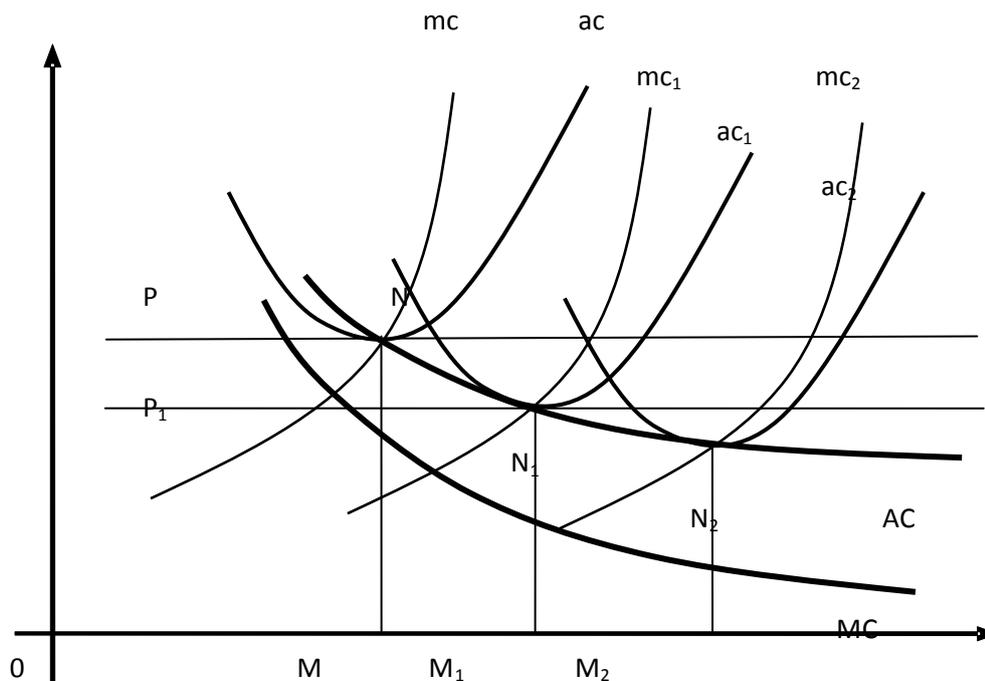
²⁵ The case of Attilio da Empoli is particularly interesting in this context as it constitutes a link between the Italian and the American discussions. The young Italian economist, does not only demonstrate already at age 27 in his *Theory of Economic Equilibrium* (1931) a great familiarity with the international discussion he is also at the University of Chicago in the years 1930-31 as a fellow of the Rockefeller Foundation. This is, of course, precisely the time during which Jacob Viner wrote and published his famous articles on "Cost Curves and Supply Curves". Unfortunately, there is no evidence of an explicit dialogue between the two economists, who spent the same time at the same University working on the same subject. Da Empoli's contribution is to render the theory of the firm dynamic in the name of economic realism. There is a striking similarity to Viner's set-up in his combination of different short-run cost curves (V-shaped rather than U-shaped due to his insistence on discontinuities) to derive a long-run cost curve with variable "fixed cost" from their envelope. For want of evidence of any interaction between da Empoli and Viner, we will assume that both writers developed the short-run/long-run interplay of a firm's cost curves independently.

²⁶ The purely algebraic representation of Luigi Amoroso (1921) with monotonously increasing marginal costs would again imply separation between variable and fixed factors.

The theoretical static long-run, it should be noted, is a sort of “timeless” long-run throughout which nothing new happens except the full mutual adjustment to each other of the primary factors existing at the beginning of the long-run period. It is more correct, therefore, to speak of long-run equilibrium in terms of the conditions which will prevail after a long-run rather than during a long-run. (Viner ([1931] 1953), 202)

These are precious clarifications. Viner goes on to state that such a long-run equilibrium remains the constantly shifting horizon of a disequilibrium process, useful mainly for indicating directions of change. Before any single long-run equilibrium is reached dynamic change will occur and imply a new long-run equilibrium. In addition, he states that such a long-run equilibrium necessarily implies the equality of marginal and average costs, since all other constellations would either induce entry or exit of firms.

Viner then applies the apparatus of the U-shaped average cost curve subsequently to the cases of increasing, constant and decreasing cost. Under the heading of increasing costs, he constructs once more the standard rising industry supply curve by adding individual rising marginal cost curves. Curiously, he neither mentions Marshall (who had already gone through the same exercise) nor Sraffa (who had strictly limited this type of reasoning to cases in which an industry uses the *whole* of a factor of production) but refers to the Ricardo’s theory of rent and his discussion of increasing cost. That is, of course, not incorrect, Marshall and Sraffa had made similar references, but it implies an originality of the discussion that it does not really possess nine years after the publication of the final edition of the *Principles* and five years after Sraffa’s 1926 article.



The development of “Constant Costs” establishes the now standard model of firm-industry interaction under competition. Viner first states correctly the somewhat esoteric case, in which constant returns to scale arise if “each producer can vary his scale of production without affecting his long-run average cost” and adds that in this case the long-run average cost curve becomes a horizontal line (Viner ([1931] 1953, 210f). However, the accompanying graph shows a series of U-

shaped average cost curves with their marginal cost curves in varying distance from the vertical axis. This would imply that different plants with different sizes (different fixed costs) would all reach their optimum scale at the same average cost – this would be an extremely unlikely coincidence by any stretch of the imagination. The second case, in which Viner considers constant returns to scale possible, is the canonical one, where all actual and potential producers would share the same minimum point of average costs,

...but with average costs increasing for each as its output increases. The long-run output of the industry would then consist of the sum of outputs of all the member concerns, each operating at that scale at which its costs are at the minimum common to all, and variations of output for the industry as a whole would result wholly from variations of the numbers of producers, each of whom would maintain a constant output... For the industry as a whole, therefore, long-run production would take place under conditions of constant long-run average and marginal cost, uniform for all producers and equal to each other, although each concern would be operating subject to short-run increasing average and marginal costs. (Viner ([1931] 1953, 212)

While this is indeed the now standard case, Viner obfuscates it slightly by insisting on the “number of producers” defining industry output. In order to avoid problems of Cournot competition, the size of the individual firm needs to be insignificant with respect to the market, i.e. the number of firm needs to be infinite. As soon as producers supply significant portions of industry output, they possess potential pricing power.²⁷ In addition, nowhere Viner mentions that long-run, industry-wide constant returns to scale would also result from establishment-level constant returns to scale, which is, as pointed out by Sraffa (see above), far from inconceivable. Finally, a combination of Viner’s first and second case would allow for changes in industry output even with a constant number of producers. Nevertheless, Viner is the first to clearly state what Sraffa had only indirectly implied that industry-level constant costs can be the result of firm-level rising costs.

Today, the most famous part of Viner’s exhaustive article is probably the section about “Net Internal Economies of Large-Scale Production”, in which he deals with the individual firm operating under long-run decreasing costs. Internal economies such as technical synergies or increased monopoly power vis-à-vis suppliers are signs of increasing returns scale, expressed by the long-run downward-sloping average cost curve AC, and hence monopoly power. Viner half-heartedly accepts this point by stating:

Provided that no change in its output will affect market price, it will pay this concern to enlarge its plant whatever the price may be, and whatever its existing scale of plant may be. If thereby it grows so large that its operations exert a significant influence on price, we pass out of the realm of atomistic competition and approach that of partial monopoly. Even then, however, it would still be profitable for this concern to enlarge its plant and increase its output as long as long-run marginal cost was lower than long-run marginal revenue... (Viner ([1931] 1953, 215)

²⁷ With producers supplying significant shares of the market, prices above the point of minimum average costs become possible, for instance, when industry demand exceeds the production at minimum average costs of a single producer but is not enough to support two producers. If markets are contestable this would lead to average cost pricing, if not, monopoly pricing results. Perhaps this is what Viner alluded to when he said that in this second case of constant costs “long-run price and output for the industry as a whole would tend to be unstable. (Viner [1931] 1953, 212)

Quite rightly, he remarks that under such conditions no definite long-run supply curve exists. However, Viner does not seem to notice that the different plant sizes corresponding to the cost-minimizing outputs M_1 and M_2 do not constitute viable equilibrium points. In fact, they only constitute transitory snap-shots of the trajectory leading to a new equilibrium. Viner could have easily avoided such ambiguities if he had stuck to his own admonition that it is “more correct... to speak of long-run equilibrium in terms of the conditions which will prevail *after* a long-run rather than *during* a long-run (see above).”

Now, it has been known since Cournot (and was reiterated by Marshall), as soon as a firm finds itself under increasing returns to scale, monopolization of the industry will ensue, which implies downward sloping demand curves. The monopolist would then choose his optimal plant size and maximize profits by equating *short-run* marginal cost with marginal revenue (his statement concerning the equalization of *long-run* marginal cost with marginal revenue is actually false). In his discussion of internal economies, he risks to throw overboard his own careful distinctions between the long-run and the short-run. Viner nowhere mentions the very different natures of his short-run and his long-run supply curves. As spelled out earlier, monotonously rising short-run cost curves express exclusively the rising costs of variable factors, whereas the downward-sloping “long-run” marginal cost curve includes in addition the progressive decrease in fixed costs. Sticking with Viner’s own original distinction and since prices move faster than plant size can adjust, short-run marginal cost is the relevant profit-maximizing variable.

This vacillation between considerations of stable equilibrium and the dynamics that lead from one equilibrium to another is also at the heart of one of the most famous errors in the history of economic thought – Viner’s instruction to his Chinese draughtsman, Dr. Y. K. Wong to draw the long-run average cost curve through the cost-minimizing points of the different short-run average cost curves and *at the same time* have it be the “envelope” of the short-run average cost curves. The former implies linking *different* equilibrium positions with one line. The latter implies choosing the optimum configuration out of *one* choice set of different combinations between fixed and variable cost, of which the envelope is the outer limit, in order to determine – in function of the demand curve – the optimum size of production.²⁸

There is great ambiguity in Viner’s reasoning who is unwilling to make the final step towards a theory of monopoly in the presence of increasing returns to scale (witness also the horizontal demand curves in the graph, the price lines P and P_1). His position at the eve of the entrance of monopolistic competition theory underlines the inevitability of the latter in order to organize the tension between monopolistic elements and competition in a more coherent manner. Viner’s concluding attack on Marshall in this context does not improve matters:

²⁸ Viner’s magnanimous acknowledgement of his error in the “Supplementary Note” to his article, unfortunately does not clear up the confusion and limits itself to humorously gloss over the deeper issues:

I do not take advantage of the opportunity to revise my 1931 article. Even the error in Chart IV [the graph reproduced above] is left uncorrected so that future teachers and students may share the pleasure of many of their predecessors of pointing out that if I had known what an “envelope” was I would not have given my excellent draftsman the technically impossible and economically inappropriate assignment of drawing an AC curve which would pass through the lowest cost points of all the ac curves and yet not rise above any ac curve at any point. (Viner [1931] 1953, 227)

Dynamic adjustment through time or stable equilibrium cannot be achieved both at the same time. However, which one of the two Viner wants to pursue in the case of increasing returns is still unclear.

To negatively-inclined long-run cost curves such as the AC and MC curves in Chart IV [the graph reproduced above], Marshall has denied the characteristic of “reversibility”, i.e., of equal validity whether output is increasing or decreasing, on the ground that some of the economies accruing when the output of a concern, or of an industry as a whole, is increased will be retained if the output of the concern or of the industry returns to its original dimension. This reasoning appears to involve confusion between static and dynamic cost curves. (Viner ([1931] 1953, 215)

Yes, Marshall has made such statements, but Marshall primarily sowed confusion by *attributing* (rather than *denying*) reversibility to long-run declining industry supply curves with the help effects external to the firm but internal to the industry. If anything, Marshall’s and Viner’s position are remarkably alike in being unable or unwilling to draw sharp dividing lines between dynamic and static considerations in the case of increasing returns to scale

Viner’s seminal contribution was to establish U-shaped average cost curves as the widely known representation of the firm’s cost function under competition and constant returns to scale as which it is known today due to his thorough and complete exposition. While several writers had developed and used the concept before him, none had explored its structure and implications to the extent Jacob Viner did. Nevertheless, even Viner’s diligence and circumspection were able to dispel entirely the intrinsic ambiguities of the subject, which remain with us every time we present the concept to the next generation of economists.

9. Conclusion

In presenting a more precise history of the development of the U-shaped average cost curve than was available until today, this essay had a triple purpose. First, establishing primacy of discovery is always a useful task in the history of economic thought, attributing due recognition to those that have been overlooked and uncovering hitherto hidden theoretical bloodlines. In the case of the U-shaped average cost curve, this task has proven far more difficult than usual, with Barone, Edgeworth and Sraffa each being able to put forward a claim to a concept that is habitually attributed either to Marshall or Viner. This traditional attribution is based on a very incomplete rendering of the historical facts. Caricaturing slightly one may state that what Marshall had to say on the subject of the U-shaped average cost curve was original but hardly pertinent; what Viner had to say on the subject was pertinent but hardly original.

The second purpose of this article is more of theoretical than of historical nature. The difficulties to attribute the concept of the U-shaped average cost curve to one single theorist in particular are anything but coincidental. They reflect the ambiguities of a concept, which allows approaching a key issue in economic theory – the equilibrium of a discretely-sized firm in a competitive industry – from a variety of different angles. At the same time, this hybrid nature also goes a long way to explain the extraordinary fecundity and popularity of the concept. Setting each subsequent step into the context of its time and motivating each theorist’s special interest, this article also shows the intrinsic ambiguities of the concept of the U-shaped average cost curve.

During the evolution of the U-shaped average cost curve, microeconomics went from Marshall to Viner. On the way, the development of the U-shaped average cost curve involved some early mathematical economics, as well as the birth of both Sraffian economics and monopolistic competition theory. The process of clarification over roughly 25 years that is covered by this essay goes hand in hand with the process in which economic theorists defined the methodological

standards against which they wanted to be judged against: logical consistency rather than descriptive realism and preferably both.

The history of the U-shaped average cost curves thus synthesizes a great part of the general development of economic science in the first three decades of the 20th century. In its realization through a joint effort by several of the greatest economists of their time, it symbolizes like no other concept the desire to reach a compromise between an analytically solid comparative statics approach and descriptive realism. The former would favor a notion of constant costs; the latter would imply notions of fixed cost and increasing returns.

The third purpose of this essay was to contribute to the gradual recognition of the important involvement of Italian economists in the economic debates of the 20th century. Their input was not limited to brilliant but isolated insights of individual geniuses but a coherent enterprise of two generations of scholars to find their way, frequently on the basis of Pantaleoni's writings, between the monuments of Vilfredo Pareto on the one hand and Alfred Marshall on the other. Not unlike Jules Dupuit and the French "engineers" in the 19th century, Enrico Barone, Luigi Amoroso, Piero Sraffa, and Attilio da Empoli form in the 20th century an important and so far insufficiently recognized strand that evolved in close and conscious interaction with the key developments in Great Britain, most notably in Cambridge. The history of the U-shaped average cost curve allows focusing their various contributions around the fundamental problem of the behavior of the individual firm under competitive conditions, one of the defining issues of economic theory.

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